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Four-dimensional construction of bulk supersymmetry breaking

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Abstract. In this letter, we discuss a four-dimensional model with modulus fields which are responsible for supersymmetry breaking. Given a non-trivial moduli dependence of the action, the model is found to give a proper description of higher-dimensional supersymmetry breaking. We explicitly calculate the gaugino and scalar mass spectrum and show that several classes of scenarios proposed in the literature are described in certain regions of the parameter space of the moduli vacuum expectation values. The model in other generic regions of the moduli space gives unexplored scenarios (mass spectra) of supersymmetry breaking in four dimensions.

1 Introduction

Supersymmetry is one of the most interesting ideas which have been introduced to overcome some unsatisfactory points of the standard model. For example, the gauge coupling unification based on precise electroweak measurements [1] and the stability of the Planck/weak mass hierarchy [2] are great successes of phenomenological applications of supersymmetry. It is, however, experimentally certain that supersymmetry is broken above the weak scale, while a variety of mechanisms for supersymmetry breaking have been proposed so far.

Among these, the mechanisms which are involved in higher-dimensional physics have been extensively studied in various ways. The existence of extra spatial dimensions provides novel ways to break supersymmetry and to communicate it to our four-dimensional world, which is the low-energy effective theory of the models. A well-known framework is string-inspired four-dimensional supergravity [3]. In large classes of these models, there are two modulus fields concerned with the compactified extra dimensions, called the dilaton and the overall modulus, which are assumed to develop non-vanishing vacuum expectation values (VEV) in their auxiliary components. The supersymmetry-breaking effect is transmitted to our low-energy degrees of freedom via (super-) gravity interactions. There have been other interesting mechanisms for supersymmetry breaking with extra dimensions [4]. These approaches provide phenomenologically viable particle spectra due to the intrinsic nature of higher-dimensional theories.

In this letter, we present a purely four-dimensional framework which can describe supersymmetry breaking in the bulk. To this end, it is convenient to regard extra dimensions as being latticized [5,6]. With this method, it is possible to revisit many interesting features of higher-dimensional effects from the four-dimensional point of view [7]. Thus, models can incorporate various four-dimensional mechanisms, such as the ones for flavor problems, and at the same time utilize the five-dimensional nature stated above. We study a model with two types of modulus fields which are supposed to have supersymmetry-breaking VEVs. Given non-trivial moduli dependences of the action, it is found that certain limits in this two-dimensional parameter space of VEVs reproduce the mass spectra of the bulk scenarios in the literature. Other generic regions of the moduli space give unexplored scenarios for supersymmetry breaking.

In Sect. 2, we explain our setup and briefly touch on the spectrum of vector fields. Supersymmetry breaking (non-zero F-terms) in this model is discussed in Sect. 3, where we identify various modulus fields and reveal their connections in the light of the construction of model. In Sect. 4, we calculate the mass spectrum of the vector multiplets with the non-vanishing moduli F-terms, and show typical mass splitting in the limits that correspond to various supersymmetry-breaking mechanisms in higher dimensions. Section 5 is devoted to a summary of our results.

2 Model

We consider a four-dimensional supersymmetric gauge theory with N copies of the gauge groups $G^N = G_1 \times G_2 \cdots \times G_N$. We assume that, for simplicity, all the G_i of

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Table 1. The matter content

	G_1	G_2	G_3	$\cdots$	G_N
Q_1	$\square$	$\bar{\square}$	1	$\cdots$	1
Q_2	1	$\square$	$\bar{\square}$	$\cdots$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
Q_N	$\bar{\square}$	1	1	$\cdots$	$\square$

the gauge theories have the same structure and particularly have a common gauge coupling g. The $N = 1$ vector multiplet V_i of the G_i gauge theory contains a gauge field A^i_μ and a gaugino λ^i. In addition, there are $N = 1$ chiral multiplets Q_i $(i = 1, \cdots, N)$ in the bifundamental representation, that is, Q_i transforms as $(\square, \bar{\square})$ under the (G_i, G_{i+1}) gauge symmetries. The fields Q_i are referred to as link variables in that they link two neighboring gauge theories. The field content of the theory is summarized in Table 1. It is shown that this simple model can imitate a five-dimensional theory with bulk gauge multiplets [5,6]. Consider the link variables Q_i developing vacuum expectation values proportional to the identity, $\langle Q_1\rangle = \cdots = \langle Q_N\rangle = v$.[1] Below the scale $\sim gv$, the gauge symmetries are reduced to a diagonal subgroup G and the other gauge multiplets become massive with discrete mass spectrum. This just looks like a five-dimensional G gauge theory compactified on a circle S^1, resulting in Kaluza–Klein mass spectra. Note here that we make the simple assumption for the bulk theory of being five-dimensional Lorentz invariant; the gauge couplings and VEVs of Q_i take common values. This way of deconstructing or of latticized dimensions is useful in that one can study higher-dimensional theories from a familiar four-dimensional point of view.

We here briefly review the mass spectrum of the gauge bosons in this model [5,6]. The complete Lagrangian and mass spectra are given later. The mass matrix is derived from the Kähler term of the Q_i fields, which gives

$$\mathcal{L} = \frac{1}{2}k^2 g^2 v^2 A^i_\mu M_{ij} A^{\mu\, j}, \tag{2.1}$$

where we have not written the implicit dependence of the gauge indices, and k is the normalization factor of the link variables that could depend on the gauge coupling g (see the Lagrangian in (4.1)). The matrix M is

$$M = \begin{pmatrix} 2 & -1 & & & -1 \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ -1 & & & -1 & 2 \end{pmatrix}. \tag{2.2}$$

From this, one obtains the mass eigenvalues m_n^2 and the corresponding eigenstates $\widetilde{A}_n$, labelled by an integer n (the Kaluza–Klein level),

$$m_n^2 = 4k^2 g^2 v^2 \sin^2 \frac{n\pi}{N}, \tag{2.3}$$

$$\widetilde{A}^n_\mu = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} (\omega_n)^j A^j_\mu \qquad (n = 0, \cdots, N-1), \tag{2.4}$$

where $\omega_n = \mathrm{e}^{2\pi \mathrm{i} n/N}$. One can see that there is a massless gauge boson and in addition a Kaluza–Klein tower of massive gauge fields, the low-lying modes ($n \ll N$) of which gauge bosons have masses approximately written as

$$m_n \simeq 2kgv\frac{n\pi}{N} \;=\; \frac{n}{R}, \tag{2.5}$$

where we identify the compactification radius as $2\pi R = N/kgv$. The mass term (2.1) becomes the kinetic energy transverse to the four-dimensions in the continuum limit ($N \to \infty$).

[1] The diagonal form of the VEVs is provided, for example, by the superpotential introduced in [5,8], which gives (supersymmetry-breaking) masses only to the trace part of Q_i, and does not affect the discussion below

3 Moduli and supersymmetry-breaking scenarios

3.1 Moduli

A supersymmetry-breaking scenario in this type of models was examined in [8,9] assuming that supersymmetry-breaking dynamics is on one endpoint of the lattice sites. From a five-dimensional viewpoint, that corresponds to supersymmetry being broken only on a four-dimensional space like the gaugino mediation [10].

In this work, we study supersymmetry breaking in the above four-dimensional model. To have insights into bulk symmetry breaking, there need to be some modulus fields which are commonly coupled to any multiplet in the theory. Here we consider two candidates of these moduli. One is the dilaton field S. One may define a modulus S_i for each gauge group whose scalar component gives a gauge coupling constant g_i. As noted before, however, they have to interact with a universal strength in order for this model to describe a proper five-dimensional theory (on the flat background). In what follows, we therefore assume $S \equiv S_1 = \cdots = S_N$. We have also assumed the universal value v for the VEVs of the link variables. Another modulus we will consider is referred to as Q; it gives this universal VEV. The modulus Q may be a normalized composite field of the Q_i. We take the modulus forms which are invariant under a translation transverse to the four dimensions, for simplicity. Non-universal values of couplings and VEVs may be interpreted as the presence of brane-like interactions and/or curved backgrounds, and that issue will be studied elsewhere.

These modulus fields may have some connections with spacetime symmetries since the modulus S corresponds to dilatation and Q relates to the size of the compactification radius. It should be noticed that, exactly speaking, S is neither a four- nor a five-dimensional dilaton, and it might not be correct to take Q to be the radion. In our model, all of these are not independent variables as will be seen below.

Let us discuss the relations between these combinations of modulus fields. First we have the dilaton S and the modulus Q whose VEVs are assumed to be

$$S = \frac{1}{4g^2} + F_S\theta^2, \tag{3.1}$$

$$Q = v + F_Q\theta^2. \tag{3.2}$$

In addition to these, we define the (combinations of) moduli fields that give the following VEVs:

$$S_4 = \frac{1}{4g_4^2} + F_{S4}\theta^2, \tag{3.3}$$

$$S_5 = \frac{1}{4g_5^2} + F_{S5}\theta^2, \tag{3.4}$$

$$T = \frac{1}{R} + F_T\theta^2, \tag{3.5}$$

where g_4, g_5 are the effective four- and five-dimensional gauge couplings, and R is the compactification radius of extra dimensions. By comparing the low-energy description of the model (at the energy below $\sim v$) with Kaluza–Klein theory, the following tree-level relations among the parameters are found [5,6][2]:

$$\frac{1}{2\pi R} = \frac{kgv}{N}, \qquad g_4^2 = \frac{g^2}{N}. \tag{3.6}$$

The first equation is required to match the spectrum to that of Kaluza–Klein theory, and the second equation can be regarded as a volume suppression of the bulk gauge coupling. In addition, the five-dimensional gauge coupling is defined (irrespectively of how to get a five-dimensional model) by

$$g_5^2 = 2\pi R g_4^2, \tag{3.7}$$

which comes from the normalization of the gauge kinetic terms. These relations among the couplings suggest that the modulus fields satisfy the relations

$$S_4 = NS, \tag{3.8}$$

$$S_5 = \frac{1}{2}QS^{1/2}k(S), \tag{3.9}$$

$$T = \frac{\pi}{N}QS^{-1/2}k(S). \tag{3.10}$$

The appropriate form of the factor $k(S)$ will be fixed in the next section by holomorphy and other phenomenological arguments. From these, we see that S_4, S_5 and T are expressed in terms of the two moduli S and Q. Of course every choice of two independent moduli such as (S, S_5), (S_4, T) et cetera can describe the same physics, and in the present four-dimensional model a natural choice is (S, Q). Each set of non-vanishing F-terms corresponds to one supersymmetry-breaking scenario.

[2] The 1PI and holomorphic gauge couplings differ only at higher-loop level in perturbation theory

Extracting the θ^2-terms, we obtain the F-components of the moduli

$$F_{S4} = NF_S, \tag{3.11}$$

$$F_{S5} = \frac{kv}{4g}\left(\frac{F_Q}{v} + 2g^2\left(1 + 2\left\langle\frac{\partial \ln k(S)}{\partial \ln S}\right\rangle\right)F_S\right), \tag{3.12}$$

$$F_T = \frac{2\pi kgv}{N} \times \left(\frac{F_Q}{v} - 2g^2\left(1 - 2\left\langle\frac{\partial \ln k(S)}{\partial \ln S}\right\rangle\right)F_S\right). \tag{3.13}$$

It is emphasized that the four-dimensional dilaton S_4 is almost close to the dilaton S, but its F-term satisfies the relation

$$\frac{F_{S4}}{\langle S_4\rangle} = \frac{F_{S5}}{\langle S_5\rangle} - \frac{F_T}{\langle T\rangle}, \tag{3.14}$$

independently of the detailed form of $k(S)$. Notice that this relation comes out through (3.7), which implies that S_4 depends on the radion T and the five-dimensional dilaton S_5.

In the next section, we will discuss supersymmetry-breaking effects of these moduli and calculate the sparticle mass spectrum of the model. When introducing appropriate potentials for the modulus fields, their VEVs are fixed to some region or point in the moduli space of vacua. For example, since S is the dilaton for each gauge group, dilaton stabilization mechanisms proposed in the literature are easily incorporated in our framework. The situation is similar for the modulus Q, corresponding to the radion. Moreover in describing five-dimensional theory, Q is actually assumed to be stabilized by relevant superpotential terms as in [5,8]. It is therefore understood that the deformation of (superpotential) terms could also induce a supersymmetry-breaking VEV of Q. However, instead of doing that, we study a more generic case. That is, in this letter we investigate the whole parameter space of the moduli F-terms, and then focus on several limits corresponding to bulk supersymmetry-breaking scenarios. We do not try to construct specific dynamics for modulus fields to have a five-dimensional nature by tuning the potential couplings, since our aim here is not to present five-dimensional theories. It is only the specific region of moduli space where our model reproduces the known bulk supersymmetry-breaking scenarios. In other words, the present framework contains unexplored four-dimensional phenomena of supersymmetry breaking. It should be noted that the tree-level mass formulae given in the next section are not modified by the existence of moduli potentials. The only possible case where the mass formula might be affected is that potentials for moduli stabilization contain the multiplets for which one wants to calculate their spectrum. We do not consider such a peculiar case in this letter.

3.2 Supersymmetry breaking in the bulk

So far, various supersymmetry-breaking models have been discussed in the literature within the frameworks of the concern with higher-dimensional physics, and several examples are mentioned in the Introduction. In the following, we will particularly focus on the dilaton and moduli dominated supersymmetry breaking in the string-inspired four-dimensional supergravity [3] and supersymmetry breaking by the radion F-term [11,12]. Here one should pay attention to relevant choices of modulus F-terms in examining supersymmetry-breaking models. That is, four-dimensional (low-energy effective) theories know F_{S4} and F_T as fundamental quantities, but on the other hand, five-dimensional ones have F_{S5} and F_T. This point is important to the following discussion.

The dilaton dominance scenario is the four-dimensional supergravity specified by a non-vanishing F-term of the four-dimensional dilaton S_4 and negligible contribution from the overall modulus T. We find from the result in the previous section that in the model where the appropriate modulus fields are S and Q, the scenario is described by $F_S \neq 0$ and $F_Q = 2g^2 v \left(1 - 2\left\langle \frac{\partial \ln k(S)}{\partial \ln S}\right\rangle\right) F_S$. The VEVs of the four- and five-dimensional dilaton F-terms are then found to be $F_{S4}/\langle S_4\rangle = F_{S5}/\langle S_5\rangle = F_S/\langle S\rangle$.

On the other hand, the moduli domination is also the four-dimensional model characterized by the opposite limit of F-terms; a non-zero F_T and a vanishing dilaton F-term, $F_{S4} = 0$. In a typical spectrum of this scenario, gauginos are massless at tree level. This is because the string perturbation theory shows that the gauge kinetic function, which induces gaugino masses, depends only on S_4 at tree level. This limit of F-terms is translated into the present model as $F_S = 0$ and $F_Q \neq 0$. The other modulus F-components are then given by $F_{S5} = (k/4g)F_Q$ and $F_T = (2\pi kg/N)F_Q$.

The field-theoretical model similar to the moduli-dominated supersymmetry breaking is discussed in [13]. This Kaluza–Klein mediation model is a four-dimensional effective theory and has the identical F-term VEVs as those in the moduli domination. Sparticle mass spectra in this case are easily calculated from renormalization-group functions in four dimensions, and the mechanism has a wide variety of realistic model construction.

A related idea utilizing F_T supersymmetry breaking is suggested in the radion mediation model. It is a five- (or higher) dimensional model, and therefore a reasonable choice of two independent moduli is T and S_5. The radion mediation is thus defined by $F_T \neq 0$ and $F_{S5} = 0$. In turn, this corresponds to $F_S \neq 0$ and $F_Q = -2g^2 v\Big(1 + 2\left\langle \frac{\partial \ln k(S)}{\partial \ln S}\right\rangle\Big)F_S$ in the present model. As a result, the four-dimensional dilaton F-term becomes

$$F_{S4} = \frac{-N}{2g^2 v}\left(1 + 2\left\langle \frac{\partial \ln k(S)}{\partial \ln S}\right\rangle\right)^{-1} F_Q = \frac{-N^2}{8\pi k g^3 v} F_T. \tag{3.15}$$

This means that we have the four-dimensional gaugino mass $m_\lambda = -F_{S4}/2\langle S_4\rangle = F_T/2\langle T\rangle$, which agrees with the result of zero-mode gaugino mass in [12].

In this way, we show via deconstruction that various known supersymmetry-breaking scenarios can be seen by the difference in the choices of non-zero modulus F-terms (as summarized in Table 2). The parameter space spanned by two independent F-terms is therefore the space of supersymmetry breaking in the bulk, and several special limits in this parameter space correspond to the scenarios which have been discussed in the literature.

4 Spectrum

In this section, we explicitly show the resulting supersymmetry-breaking spectrum of Kaluza–Klein modes. We here focus on the vector multiplets, but the quantitative aspects discussed below are completely the same for bulk hypermultiplets.

Since we consider broken gauge symmetries and massive gauge fields, it is convenient to use the unitary gauge for vector multiplets. In this gauge, the Goldstone chiral multiplets (the fluctuations around the VEVs (3.2)) are absorbed into the vector multiplets with suitable gauge transformations. Consequently each vector multiplet contains a massive vector field and two spinors, a gaugino and a Goldstone fermion. In addition, other dynamical and auxiliary bosonic components are introduced. The link variables Q_i are then treated as background fields with non-zero VEVs.

First it is easily found that the gauge fields do not get a supersymmetry-breaking contribution, and the mass spectrum is just given by the one calculated in Sect. 2; one massless gauge multiplet corresponding to the diagonal subgroup G and the Kaluza–Klein tower with discrete mass spectrum (2.3).

The gauge fermion masses with supersymmetry breaking are calculated as follows. The relevant piece of the Lagrangian is

$$\mathcal{L} = \sum_i \left[\int \mathrm{d}^2\theta\, S W_i W_i + \mathrm{h.c.} + \int \mathrm{d}^2\theta \mathrm{d}^2\bar\theta\, K(S,S^\dagger)\, Q_i^\dagger \mathrm{e}^{\sum V} Q_i\right]. \tag{4.1}$$

We have included the universal couplings of the dilaton S. The relevant field to appear here is S, and not the effective four- or five-dimensional dilaton S_4, S_5. The real function $K(S,S^\dagger)$ fixes the overall scale of the discrete mass spectra of this model ($k = \langle K|_{\theta=0}\rangle^{1/2}$) and its form will be determined later. Inserting the VEVs of (3.1) and (3.2), the mass terms take the following form:

$$\mathcal{L}_{\mathrm{mass}} = -F_S \lambda_i \lambda_i - k^2 v^2 \chi_i M_{ij} \lambda_j + \frac{1}{2} k^2 v \left(F_Q + v\left\langle \frac{\partial \ln K(S,S^\dagger)}{\partial \ln S}\right\rangle F_S\right) \chi_i M_{ij} \chi_j + \mathrm{h.c.}, \tag{4.2}$$

where χ_i is the Goldstone fermion now included in the vector multiplet V_i. The first term comes from the gauge kinetic term and the last two are induced by the tree-level

Table 2. The moduli F-terms and the typical supersymmetry-breaking models in the bulk. The parameter x is defined by $x \equiv 2\left\langle \frac{\partial \ln k(S)}{\partial \ln S} \right\rangle$. The holomorphy and some phenomenological arguments suggest $x = 1$ and $k = 1/g$

	F_S	F_Q	F_{S4}	F_{S5}	F_T
dilaton ($F_T = 0$)	F_S	$2g^2v(1-x)F_S$	NF_S	$kgvF_S$	0
moduli ($F_{S4} = 0$)	0	F_Q	0	$\frac{k}{4g}F_Q$	$\frac{2\pi kg}{N}F_Q$
radion ($F_{S5} = 0$)	$\frac{-F_Q}{2g^2v(1+x)}$	F_Q	$\frac{-NF_Q}{2g^2v(1+x)}$	0	$\frac{4\pi kg}{N(1+x)}F_Q$

Kähler term of Q_i, so the flavor structure is the same as that of the gauge fields, which is explained by the matrix M; see (2.2). Since M also defines the kinetic terms of the χ_i, the canonically normalized fields are obtained by the redefinition $kvP\chi \to \chi$ where P is a square-root of M ($M = P^{\mathrm{t}}P$) and we can write

$$P = \begin{pmatrix} 1 & -1 & & \\ & \ddots & \ddots & \\ & & 1 & -1 \\ -1 & & & 1 \end{pmatrix}. \tag{4.3}$$

With this redefinition and a rescaling $\lambda \to g\lambda$, the mass matrix of the normalized spinors becomes

$$\mathcal{L}_{\mathrm{mass}} = -\frac{1}{2}\begin{pmatrix}\lambda & \chi\end{pmatrix}\begin{pmatrix} 2g^2F_S & kgvP^{\mathrm{t}} \\ kgvP & -\frac{F_Q}{v} - \left\langle \frac{\partial \ln K(S,S^\dagger)}{\partial S}\right\rangle F_S \end{pmatrix}\begin{pmatrix}\lambda \\ \chi\end{pmatrix} + \mathrm{h.c.} \tag{4.4}$$

Without the F-term contributions, the mass eigenstates take the same form as the gauge fields. This is an indication of $N = 2$ supersymmetry; equivalently: $N = 1$ supersymmetry in five dimensions, which is expected to appear in the infrared. In this mass basis of $\widetilde{\lambda}_n$ and $\widetilde{\chi}_n$, the mass matrix is rewritten as follows:

$$\mathcal{L}_{\mathrm{mass}} = -\frac{1}{2}\begin{pmatrix}\widetilde{\lambda} & \widetilde{\chi}\end{pmatrix}\begin{pmatrix} 2g^2F_S & B \\ B & -\frac{F_Q}{v} - \left\langle \frac{\partial \ln K(S,S^\dagger)}{\partial S}\right\rangle F_S \end{pmatrix}\begin{pmatrix}\widetilde{\lambda} \\ \widetilde{\chi}\end{pmatrix} + \mathrm{h.c.}\,, \tag{4.5}$$

where the elements of the diagonal matrix $B_{ij} = 2kgv \times \sin\frac{j\pi}{N}\,\delta_{ij}$ are the Kaluza–Klein Dirac masses. The irrelevant phase factors have been absorbed in the field redefinitions. We finally obtain the mass eigenvalue of the level-n Kaluza–Klein gauge fermions ($n = 0, \cdots, N-1$);

$$m_{\lambda_n} = \frac{1}{2}\left[\pm\sqrt{4m_n^2 + \left(\frac{F_Q}{v} + 2g^2\left(1 + 2\left\langle\frac{\partial \ln K(S,S^\dagger)}{\partial \ln S}\right\rangle\right)F_S\right)^2} - \frac{F_Q}{v} + 2g^2\left(1 - 2\left\langle\frac{\partial \ln K(S,S^\dagger)}{\partial \ln S}\right\rangle\right)F_S\right], \tag{4.6}$$

where m_n is the Kaluza–Klein mass of the gauge fields (2.3), which is a supersymmetric contribution. The positive (negative) sign in the bracket corresponds to the gaugino (the Goldstone fermion) mass. Here the states which are equal to $\widetilde{\lambda}$, $\widetilde{\chi}$ in the supersymmetric limit are referred to as gauginos and Goldstone fermions, respectively. It is interesting to note that the gauge fermion mass (4.6) can be more simply expressed with only the five-dimensional quantities:

$$m_{\lambda_n} = \frac{1}{2}\left[\pm\sqrt{4m_n^2 + \left(\frac{F_{S5}}{\langle S_5\rangle}\right)^2} - \frac{F_T}{\langle T\rangle}\right]. \tag{4.7}$$

This result implies that higher-dimensional effects, even including supersymmetry breaking, are properly reproduced in our model.

We now examine our result for the supersymmetry-breaking models discussed in the previous section.

4.1 Dilaton dominated supersymmetry breaking

This scenario is characterized by the limit $F_T = 0$. We then obtain the Kaluza–Klein masses with the supersymmetry-breaking effect

$$m_{\lambda_n}(\mathrm{dilaton}) = \pm\sqrt{m_n^2 + (2g^2F_S)^2}. \tag{4.8}$$

This spectrum is just as expected in the dilaton dominant case in supergravity models. The first term in the square-root is the Kaluza–Klein Dirac mass, and the second one is a supersymmetry-breaking part that is certainly provided by the four-dimensional dilaton coupling ($2g^2F_S = F_{S4}/2\,\langle S_4\rangle$). Note that all the Kaluza–Klein states including zero modes receive the universal supersymmetry-breaking contribution. The two level-n spinors are degenerate in mass, and the mass splitting between bosons and fermions are equal for all Kaluza–Klein modes. This fact is regarded as a reflection of the dilaton field (the action of dilatation) commonly coupling to any field in the theory.

The universal mass spectrum is one of the major motivations to investigate the dilaton dominant limit in supergravity models. The universality in our model is more clearly seen for scalar components in hypermultiplets. In that case, taking the $F_T = 0$ limit washes away the bulk mass dependence of the supersymmetry-breaking scalar masses [14].

4.2 Moduli dominated supersymmetry breaking

With the definition of $F_{S4} = 0$, the gauge fermion mass spectrum becomes

$$m_{\lambda_n}(\mathrm{moduli, KK}) = \pm\sqrt{m_n^2 + \left(\frac{F_Q}{2v}\right)^2} - \frac{F_Q}{2v}. \quad (4.9)$$

It is interesting that even when supersymmetry breaking is turned on, the zero-mode gaugino is massless and does not get a mass splitting with the zero-mode gauge field. (The $n = 0$ spinor being affected by the non-zero F-terms is the Goldstone fermion $\widetilde{\chi}_0$.) This is exactly the spectrum predicted in this type of supersymmetry-breaking scenario [3,13]. By definition, the scenario assumes a vanishing F-term of the four-dimensional dilaton. The zero-mode gaugino mass is then shifted at loop level by string threshold corrections or by the effects of bulk fields. In our model, the spectrum is easily read from the mass matrix (4.5). The gaugino $\widetilde{\lambda}_0$ is massless due to the vanishing F_S and the Kaluza–Klein mixing mass. As for the excited modes, the supersymmetry-breaking contribution from F_Q is transmitted to gauginos through the non-zero Kaluza–Klein masses. The situation is similar to the models where gauge multiplets behave as messengers, and sparticle soft masses at loop level are calculated from wave-function renormalization in four dimensions [15]. Therefore our approach is also likely to describe this limit well.

There may be an intuitive explanation for this type of spectrum as was discussed in [13]. That is, a non-zero F-term of the modulus which gives the Kaluza–Klein masses does not induce tree-level supersymmetry-breaking masses for zero modes, as these two mass terms are proportional to the Kaluza–Klein numbers. In the present case, such a modulus corresponds to the one whose scalar component obtains a VEV $\propto 1/R$, and is given by $T \propto Q$. This interpretation becomes manifest in examining the mass spectra of bulk hypermultiplets with moduli fields [14].

4.3 Radion F-term breaking

This scenario takes the F-term assumption $F_{S5} = 0$, that is converted into $F_Q = -2g^2 v \left(1 + 2\left\langle \frac{\partial \ln k(S)}{\partial \ln S}\right\rangle\right) F_S$. We find that the gaugino mass matrix (4.5) in this limit has exactly the same form as calculated in [16], where one uses an $N = 1$ superfield formalism of the five-dimensional action with the radion superfield. The mass eigenvalues of the Kaluza–Klein spinors become

$$m_{\lambda_n}(\mathrm{radion}) = \pm m_n - \frac{F_Q}{v} - \left\langle \frac{\partial \ln k(S)}{\partial S}\right\rangle F_S. \quad (4.10)$$

This scenario assumes a non-zero value of the radion F-term. However, compared to the moduli dominance scenario stated above, there is a difference in the contribution from the dilaton field S, resulting in the non-zero F-term of the four-dimensional dilaton S_4. This gives a tree-level mass of the gaugino zero mode. In other words, if the moduli domination were seen from a five-dimensional viewpoint, there would appear to be an additional contribution from S_5 such that the definition $F_{S4} = 0$ is preserved (see (3.14)). On the other hand, the masses of the Kaluza–Klein excited modes are rather similar to each other. In particular, the low-lying modes have masses

$$m_{\lambda_n}(\mathrm{moduli, KK}) = m_{\lambda_n}(\mathrm{radion}) \simeq \pm\frac{n}{R} - \frac{R}{2}F_T, \quad (4.11)$$

where we have assumed that the supersymmetry-breaking part is smaller than the supersymmetric Kaluza–Klein mass (i.e., $RF_T \ll v$).

It has been shown [16,17] that the radion mediation model has the same spectrum as that predicted by the Scherk–Schwarz mechanism [18]. The Scherk–Schwarz theory is essentially higher dimensional and adopts twisted boundary conditions for the bulk fields along the extra dimensions. On the other hand, the moduli-dominated supersymmetry breaking in four-dimensional supergravity (and the Kaluza–Klein mediation) is not a Scherk–Schwarz theory and does give different soft terms, as explicitly shown in the above.

Let us finally discuss the normalization function $K(S, S^\dagger)$ in the Lagrangian (4.1). It should be mentioned that the form of the gaugino masses (4.7) is not affected by any details of the factor $K(S, S^\dagger)$, and the above qualitative discussions about the gaugino mass spectrum are generic and still preserved. We propose that the proper form of K is given by

$$K(S, S^\dagger) = \frac{8}{1/S + 1/S^\dagger}. \quad (4.12)$$

The factors k and $k(S)$ defined in Sect. 2 then become $k = 1/g$ and $k(S) = 2S^{1/2}$, respectively. Though the complete form of K is not determined without referring to higher-dimensional physics, (4.12) is found to be certainly consistent with several non-trivial and independent requirements. First, notice that to have right results based on holomorphy, the normalization of the link variables Q_i is required to be $\langle K\rangle = 1/g^2$. With this choice, the gauge and adjoint chiral multiplets of the low-energy G gauge theory have the same field normalization. Moreover, in this case, the radion superfield in our model becomes independent of the dilaton superfield (see the relation (3.10)), which result is plausible since, for example, it does not lead to an undesirable relation between the theta angle and the graviphoton field.

Secondly, with an explicit form of $K(S, S^\dagger)$, one can evaluate tree-level masses of the scalar fields of the Q_i. They are the adjoint scalar fields of the low-energy G

gauge theory, and are contained in vector multiplets of the enhanced $N = 2$ supersymmetry. The scalar mass m_c^2 generated by the Kähler term with (4.12) is

$$m_{c_n}^2 = m_n^2 + 2\,\mathrm{Re}\left(\frac{F_{S5}^*}{\langle S_5\rangle}\frac{F_T}{\langle T\rangle}\right). \tag{4.13}$$

Let us examine this mass formula in the limits discussed before. One can easily see that the radion mediation limit ($F_{S5} = 0$) does not give a supersymmetry-breaking soft mass. This indeed agrees with the fact that the radion mediation is equivalent to the Scherk–Schwarz mechanism, which is now applied to the $SU(2)_\mathrm{R}$ symmetry under which the adjoint scalars are singlet and hence do not get symmetry-breaking masses. If one first requires that the scalars c_n do not have soft terms in the $F_{S5} = 0$ limit, $K(S, S^\dagger)$ has to satisfy $\left\langle \frac{\partial \ln K}{\partial S \partial S^\dagger}\right\rangle = -(2g^2)^2$. The most probable solution of this equation is $K = X(S)X(S^\dagger)/(S + S^\dagger)$, where X is an arbitrary function. Then the holomorphy argument suggests $X(S) \propto S$ and thus (4.12). For completeness, we write down the scalar masses in the other limits:

$$m_{c_n}^2(\text{dilaton}) = m_n^2,$$
$$m_{c_n}^2(\text{moduli, KK}) = m_n^2 + 2\left|\frac{F_T}{\langle T\rangle}\right|^2. \tag{4.14}$$

The third consistency concerns the 5-5 component of the five-dimensional metric, g_{55}. In a continuum five-dimensional theory, the kinetic energy terms of bosonic fields along the fifth dimension have a dependence on g_{55} as $\sqrt{g_{55}}\, g^{55} \propto 1/R$. In the model at hand, the second term in the Lagrangian (4.1) becomes this kinetic energy in the continuum limit, and its modulus dependence is given by $\left\langle K(S, S^\dagger)\, Q^\dagger Q\right\rangle$. Equation (4.12) then indicates $\left\langle KQ^2\right\rangle \sim \left\langle SQ^2\right\rangle \sim \langle S_5 T\rangle$. As a result, the desirable metric dependence appears, for a fixed value of the five-dimensional gauge coupling g_5.

We close this section by a comment on the model which turns into a five-dimensional theory compactified on an S^1/Z_2 orbifold. This can be formulated [6,8] by getting rid of a link variable, e.g. Q_N, from the S^1 model. In this case, additional fields may be introduced to cancel the gauge anomalies on the orbifold fixed points. Examining a mass matrix, it is found that the Q_i contain only massive modes, and the zero-mode state consists of an $N = 1$ vector multiplet without an associated adjoint chiral multiplet, which situation corresponds to the Z_2 orbifolding. In turn, this results in removing $\widetilde{\chi}^0$ and c_0 in our analyses. The plus sign is chosen for the zero mode, and the gaugino masses in various limits discussed before are not altered. Results similar to those as in the S^1 case hold for the other quantities; for example, the Kaluza–Klein mass spectrum is unchanged except for replacing $N \to 2N$ ($R \to 2R$).

5 Summary

In this paper, we have studied supersymmetry breaking in the four-dimensional model with two types of modulus fields. The model can describe five-dimensional physics in the infrared, and given the relations among the modulus fields, we have discussed supersymmetry breaking in the higher-dimensional bulk. The analysis is based on a four-dimensional model, that is renormalizable and calculable in a usual manner. We have made it clear that several specific limits in the two-dimensional parameter space of the modulus F-terms correspond to the bulk supersymmetry-breaking scenarios in the literature. We have shown this by examining the gaugino and adjoint scalar masses in the cases of the S^1 and S^1/Z_2 compactifications. It is non-trivial to establish such correspondences and indeed require a properly fixed moduli dependence of the action. The moduli dependence will also be confirmed by detailed calculations of radiative corrections to the mass spectrum [14]. Moreover it would be an interesting issue to study other choices of the couplings and limits, which could describe unexplored supersymmetry breaking in four or higher dimensions, and we leave this to future work. Besides the issue of supersymmetry breaking, extra dimensions provides a new perspective for various subjects in particle physics. Realistic model construction along this line of using a purely four-dimensional one will deserve further investigations.

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